**CHEN/ SENG 460/660**

**Homework 4 (SOLUTION)**

1. Consider a pump following a Weibull failure distribution with a shape parameter of 1.4 and a scale parameter, characteristic life, of 550 days.
2. State the behavior range represented by this system (DFR, CFR, or IFR) and state your reason.

Answer:

Expected behavior is IFR, because the shape parameter, β>1

1. If two of these pumps are made redundant by connection in parallel, calculate the reliability of the system of two pumps at 100 days, R(100). (Hint: Unit 7C, Slide 23)



1. What is the probability of failure of the system at 100 days?

F(t) = 1-R(t)=1-0.992

1. Write the expression for and find the maximum number of Weibull components, assumed identical and independent, with scale parameter of 10,000 operating hrs and a shape parameter of β = 1.8 to be placed in series to meet a Reliability criterion of 0.95 at t = 100 operating hr. State what region of the bathtub curve represents the behavior of this component, and state why.

Answer:

Behavior is IFR, increasing failure rate region of the Bathtub curve, because β > 1.

R(t)=→ R(100)=

units

MTTF=

1. Burn-in testing is the process by which early failures in components are detected and removed, thereby increasing component reliability. During burn-in, the component is exercised under extreme operating conditions (elevated temperatures and voltages). This stresses the component under test and eliminates the “weak” population from the product prior to customer delivery. Burn-in testing attempts to weed out failures from the ‘burn-in’ (DFR) stage of the “bathtub” curve.

A component has a Weibull failure distribution with β = 0.86, and its characteristic life is 2450 days. By how many days will the design life for a 0.90 reliability specification be extended as a result of a 30-day burn-in period?

Begin with an expression for the Conditional Reliability to work an additional time t beyond t0 = 30 days (the burn-in period) given the probability of working at t0 = 30, and solve for t. Then compare this t with the design life without a burn in period. When you have calculated the amount of extension in the design life, comment on the expected cost effectiveness of the burn-in period to extend the 0.90 Reliability specification.



1. Five identical road graders are used for a highway project. Company data show that the operational life T of each grader follows Lognormal behavior with a mean life of 1500 hr, σ =450 hr, and δ or cov (Coefficient of Variation) = σ/μ = 0.30. Assuming independence among the machines, you want to calculate the probability that 2 of the 5 graders will fail within 900 hr of operation.

[Note that:

1. we can use the Bernoulli sequence to represent the two states of machine (failure or success), because the 3 conditions are approximately: binary states (success or fail), equal probabilities for each trial, and independent trials. Let the probability of one Bernoulli sequence be p = P(fail) for one machine.
2. The 2 failed machines could be any 2 of the 5 machines, so the Binomial coefficient is needed to include all Bernoulli sequences that contribute to the probability without regard to the order of failures and order of non-failures]
3. Given the 5 machines, state the expression for a Bernoulli sequence that represents the probability of 2 machines fail, 3 machines succeed.



1. State the Binomial Coefficient expression and calculate its value for this case for the number of ways 2 out of 5 machines can fail.



1. Assuming Lognormal failure behavior of the machines, write the expression for and calculate the Lognormal shape parameter, s.



So the lognormal shape parameter s=0.3

1. Write the expression for and calculate the median life value from the data mean life value. The median is needed as a location parameter for the Lognormal distribution. State why the median that you calculate is expected to be different in value from the mean.
2. Finally, write the logic expression for and calculate F(T = 900). Include in your solution a sketch of the Lognormal f(t), pdf, showing (by highlighting) the area under the curve that represents the cumulative probability of failure up to 900 hr. This failure probability (F(T=900) is the p needed in the Binomial calculation for P(x = 2).



1. Now write the expression for the Binomial distribution and calculate the probability of two failures, P(x = 2), among the 5 available machines. Use the F(t = 900) as the Binomial p value for failure. Also, write the expressions for and calculate the Mean value, Variance, Std. Dev., and cov for the Binomial pmf.





1. Write the Binomial expression for and calculate the Reliability of the system of Five Machines based on the probability of working or 0 failures at 900 hr = R(t = 900).



1. Your firm manufactures small motors with failure behavior following a Normal distribution with an average life of 10 yr (aleatory uncertainty) with a standard deviation (epistemic uncertainty) of 2 yr. Your firm has decided to replace only 3% of the motors that fail, so the appropriate warranty period beginning at t = 0 must be determined for 3% failures, so 0.03 is the probability of failure during the warranty period. Given this information, find the approximate warranty time the firm should set.



